Broken Pairs and High-Spin States in Transitional Nuclei

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Abstract

The Interacting Boson plus broken-pairs Model has been used to describe high-spin bands in spherical and transitional nuclei. In the spherical nucleus ^{104}Cd the model reproduces the structure of high-spin bands built on a vibrational structure. Model calculations performed with a single set of parameters reproduce ten bands in ^{136}Nd, including the two dipole high-spin bands based on the $(\pi h_{11/2})^2$ $(\nu h_{11/2})^2$ configuration.

1 Introduction

Models that are based on the Interacting Boson Approximation provide a consistent description of low-spin nuclear structure in spherical, deformed and transitional nuclei. By including selective non-collective fermion degrees of freedom, through successive breaking of correlated S an D pairs, the IBM can be extended to describe the physics of high-spin states in nuclei. This extension of the model is especially relevant for transitional regions, where single-particle excitations and vibrational collectivity are dominant modes and no pronounced axis for cranking exists. We present a model that extends the IBM-1 to include two- and four-fermion noncollective states (one and two broken-pairs). The model has been applied in the description of high-spin states in the Hg [1, 2, 3], Sr-Zr [4, 5, 6, 7], and Nd-Sm [8, 9, 10] regions.
The model is based on the IBM-1 [11]; the boson space consists of s and d bosons, with no distinction between protons and neutrons. To generate high-spin states, the model allows one or two bosons to be destroyed and non-collective fermion pairs formed, represented by two- and four-quasiparticle states which recouple to the boson core. High-spin states are described in terms of broken pairs. The model space for an even-even nucleus with $2N$ valence nucleons is

$$\begin{aligned} \mid N\ bosons\ > \quad \oplus & \quad \mid (N-1) bosons \otimes 1\ broken\ pair > \\ \oplus & \quad \mid (N-2) bosons \otimes 2\ broken\ pairs > \oplus\ \ ... \end{aligned}$$

The model Hamiltonian has four terms: the IBM-1 boson Hamiltonian, the fermion Hamiltonian, the boson-fermion interaction, and a pair breaking interaction that mixes states with different number of fermions.

$$H = H_B + H_F + V_{BF} + V_{mix}. \tag{1}$$

The fermion Hamiltonian H_F contains single-fermion energies and fermion-fermion interactions. The interaction between the unpaired fermions and the boson core contains the dynamical, exchange and monopole interactions of the IBFM-1 [12] In order to describe dipole bands in transitional nuclei, we have modified the quadrupole-quadrupole dynamical interaction

$$V_{dyn} = \Gamma_0 \sum_{j_1 j_2} (u_{j_1} u_{j_2} - v_{j_1} v_{j_2}) \langle j_1 \parallel Y_2 \parallel j_2 \rangle \times \left([a^\dagger_{j_1} \times \tilde{a}_{j_2}]^{(2)} \cdot Q^B \right), \tag{2}$$

The standard boson quadrupole operator Q^B has been extended by a higher order term

$$\chi' \sum_{L_1 L_2} \left[\left[d^\dagger \times \tilde{d} \right]^{(L_1)} \times \left[d^\dagger \times \tilde{d} \right]^{(L_2)} \right]^{(2)} , \tag{3}$$

The terms H_B, H_F and V_{BF} conserve the number of bosons and the number of fermions separately. In our model only the total number of nucleons is conserved, bosons can be destroyed and fermion pairs created, and vice versa. In the same order of approximation as for V_{BF}, the pair breaking interaction V_{mix} which mixes states with different number of fermions, conserving the total nucleon number only, reads

$$\begin{aligned} V_{mix} \;\; = \;\; & -U_0 \left\{ \sum_{j_1 j_2} u_{j_1} u_{j_2} (u_{j_1} v_{j_2} + u_{j_2} v_{j_1}) \langle j_1 \parallel Y_2 \parallel j_2 \rangle^2 \frac{1}{\sqrt{2 j_2 + 1}} \left([a^\dagger_{j_2} \times a^\dagger_{j_2}]^{(0)} \cdot s \right) + \; h.c. \right\} \\ & -U_2 \left\{ \sum_{j_1 j_2} (u_{j_1} v_{j_2} + u_{j_2} v_{j_1}) \langle j_1 \parallel Y_2 \parallel j_2 \rangle \left([a^\dagger_{j_1} \times a^\dagger_{j_2}]^{(2)} \cdot \tilde{d} \right) + \; h.c. \right\} . \end{aligned} \tag{4}$$

If mixed proton-neutron configurations are included in the fermion model space, i.e. there can be both proton and neutron broken pairs, the full model Hamiltonian reads

$$H = H_B + H_{\nu F} + H_{\pi F} + H_{\nu BF} + H_{\pi BF} + H^{mix}_{\nu} + H^{mix}_{\pi} + H_{\nu\pi}, \tag{5}$$

where the proton-neutron interaction term is defined

$$\begin{aligned} H_{\nu\pi} \;\; = \;\; & \sum_{nn'pp'} \sum_{J} (-)^J h_J(nn'pp') (u_n u_{n'} - v_n v_{n'}) (u_p u_{p'} - v_p v_{p'}) \times \\ & \times \left(\left[a^\dagger_n \times \tilde{a}_{n'} \right]^{(J)} \cdot \left[a^\dagger_p \times \tilde{a}_{p'} \right]^{(J)} \right) . \end{aligned} \tag{6}$$

In the following two sections we present results for quadrupole bands in the spherical nucleus ^{104}Cd, and for proton-neutron dipole bands in the transitional nucleus ^{136}Nd.

2 The nucleus $^{104}_{48}\mathrm{Cd}_{56}$

The Cd isotopes, with only two protons from the closed shell at N=50, present an example of spherical vibrational nuclei in which also high-spin quadrupole bands are found. In particular, in the nucleus ^{104}Cd quadrupole bands extend up to spin 26 $\hbar$. Here we present a description of the structure of the excitation spectrum in the framework of the IBM plus one broken pair.
In general most of the parameters of the Hamiltonian are taken from analyses of the low- and high-spin states in the neighboring even and odd nuclei. For ^{104}Cd the parameters of the boson Hamiltonian are: $\epsilon = 0.658$ MeV, $C_4 = 0.117$ MeV, the number of bosons is $N = 4$. ϵ corresponds to the excitation energy of 2_1^+, and C_4 is adjusted to reproduce the 6_1^+ and 8_1^+ states. Since only the yrast sequence of the collective vibrational structure is known experimentally, the remaining parameters of the boson Hamiltonian could not be determined, and are set to zero.
The resulting SU(5) vibrator spectrum displays very little anharmonicity. In the present calculation we only consider collective states and two-quasiparticle states based on configurations with two neutrons in the broken pair. States based on the proton $(g_{9/2})^2$ configuration are not included in the model space. The single-quasiparticle neutron energies and occupation probabilities are obtained by a simple BCS calculation. The quasiparticle energies and occupation probabilities are: $E(\nu d_{5/2}) = 1.113$ MeV, $E(\nu s_{1/2}) = 2.287$ MeV, $E(\nu h_{11/2}) = 2.691$ MeV, $E(\nu g_{7/2}) = 1.316$ MeV, $v^2(\nu d_{5/2}) = 0.57$, $v^2(\nu s_{1/2}) = 0.06$, $v^2(\nu h_{11/2}) = 0.04$, $v^2(\nu g_{7/2}) = 0.23$.
The parameters of the boson-fermion interactions have been adjusted to reproduce the lowest positive and negative parity structures in the neighboring odd-N isotopes ^{103}Cd and ^{105}Cd. For neutron states the boson-fermion parameters are: $\Gamma_0 = 0.2$ MeV and $\chi = -0.9$ for the dynamical interaction, and $\Lambda_0 = 0.2$ MeV for the exchange interaction. The parameter of the boson quadrupole operator, $\chi = -0.9$, is adjusted to reproduce the experimental data for ^{106}Cd: $B(E2, 2_1 \rightarrow 0_1) = 0.068\ e^2b^2$ and $Q(2_1) = -0.25\ eb$. In addition to the boson and boson-fermion parameters that have been already discussed, the strength parameter of the pair-breaking interaction is $U_2 = 0.1$ MeV. Results of model calculations for positive parity states in ^{104}Cd, with a fermion space of one neutron broken pair, are shown in Fig. 1.
In this energy level diagram, only few lowest calculated levels of each spin are compared to the experimental counterparts. The collective vibrational structure remains yrast up to angular momentum $I = 6^+$. In the experimental spectrum one finds, below the collective $I = 8^+$ (3211 keV) state, two other states of the same angular momentum: at 2902 keV and 3031 keV. They are probably based on a two-neutron configurations. It is interesting that this triplet of $I = 8^+$ states is also reproduced by our calculation. In fact, for three $\Delta I = 2$ positive parity sequences, based on the $(d_{5/2})^2$, $(g_{7/2})^2$ and $(d_{5/2}, g_{7/2})$ neutron configurations, probable experimental counterparts are observed. The calculated energy spacings correspond to the collective vibrational sequence. The experimental energy spacings are somewhat smaller, indicating a stronger core polarization and/or change of deformation. This is probably caused by admixtures of 2p-2h proton configurations, an effect that could not be included in our model space. In heavier Cd isotopes experimental data exists on

the neutron $(h_{11/2})^2$ structure. The $I = 10^+$ band-head of the $\Delta I = 2$ $(h_{11/2})^2$ sequence is at: 4153 KeV in ^{108}Cd, and 4816 keV in ^{106}Cd. In ^{104}Cd this state is not observed, our calculations place it at 5310 keV. ^{104}Cd has also less particles outside the closed shell compared to heavier isotopes, and therefore collective properties are less pronounced. In particular, with only four bosons we can only construct states up to angular momentum $I = 16^+$, including the fermion space of two neutrons. States with higher angular momenta should be based on a different core, probably one that includes proton excitations across the $Z = 50$ shell closure.

The lowest calculated negative-parity two neutron states are compared with experimental levels in Fig. 2. The parameters of the Hamiltonian have the same values as in the calculation of positive-parity states. There are several negative parity sequences in the experimental spectrum which start at ≈ 4 MeV, and with angular momenta $I = 7^- - 9^-$. The lowest two calculated levels of each spin $I \geq 7^-$ are displayed in Fig. 2. The levels are grouped into sequences according to the dominant fermion component in the wave function. The two-fermion angular momentum is approximately a good quantum number. The collective part of the wave functions correspond to that of the low-lying collective vibrational sequence. Because the quasiparticle energies of $d_{5/2}$ and $g_{7/2}$ differ by only ≈ 200 keV, both the $(h_{11/2}, d_{5/2})$ and the $(h_{11/2}, g_{7/2})$ configurations form sequences with states of the same angular momentum at almost the same excitation energies. Of course the density of negative parity states between 4 MeV and 7 MeV is very high. All four neutron orbitals are included in the calculation, and one finds many states with low angular momenta, that are not observed in experiment. Therefore in Fig. 2 we only display states for which correspondence with experimental data can be established.

3 The nucleus $^{136}_{60}$Nd$_{76}$

Nuclei in the A = 130-140 mass region are γ-soft and the polarizing effect of the aligned nucleons induces changes in the nuclear shape. Because of the different nature of the excitations (particles for proton, and holes for neutron configurations), the alignment of a pair of $h_{11/2}$ protons induces a prolate shape, whereas the alignment of a neutron pair in the $h_{11/2}$ orbital drives the nucleus towards a collective oblate shape. In ^{136}Nd one therefore expects to observe different coexisting structures at similar excitation energies. The IBM model with broken pairs has been previously used in the description of low-spin and high-spin properties of γ-soft nuclei of this region (in the IBM language O(6) nuclei): ^{138}Nd [8], ^{137}Nd [10] and ^{139}Sm [9].

In the present work we use the IBM with proton and neutron broken pairs to describe the excitation spectrum of ^{136}Nd. In particular, we want to obtain a correct description of high-spin dipole bands, which have been interpreted as two proton - two neutron structures. The experimental level scheme of positive-parity states is displayed in Fig. 3. The labels of bands are from Ref. [14]; in addition to the ground state band and the quasi γ-band, bands 3, 5, 7 and 8 result from the alignment of two protons or two neutrons in the $h_{11/2}$ orbital, the two four-quasiparticle dipole bands have labels 10 and 11.

There are 6 neutron valence *holes* and 10 proton valence *particles*. The resulting boson number is N=8. The set of parameters for the boson Hamiltonian is: ϵ=0.36, C_0=0.16, C_2=-0.12, C_4=0.19, V_2=0.11 and V_0=-0.3 (all values in MeV). The boson parameters have values similar to those that have been used in the calculation of ^{138}Nd, ^{137}Nd and ^{139}Sm.
In A $\approx$ 140 nuclei the structure of positive parity high-spin states close to the yrast line is characterized by the alignment of both proton and neutron pairs in the $h_{11/2}$ orbital. For positive-parity states we have only included the proton and neutron $h_{11/2}$ orbitals in the fermion model space. Additional single-nucleon states make the two broken-pairs bases prohibitively large. The single quasiparticle energies and occupation probabilities are obtained from a BCS calculation. Similar to our previous calculations for ^{138}Nd and ^{137}Nd, the resulting quasiparticle energies for the proton and neutron $h_{11/2}$ states had to be slightly renormalized. $E_\nu(h_{11/2}) = 1.75$ MeV, $v_\nu^2(h_{11/2}) = 0.83$, $E_\pi(h_{11/2}) = 1.60$ MeV, $v_\pi^2(h_{11/2}) = 0.07$. In order to further reduce the large size of the space with two-broken pairs, we had prediagonalized the boson Hamiltonian, and the fermion states were then coupled to the lowest eigenvectors, i.e. only to the collective ground state band. The parameters of the fermion-boson interactions are determined from IBFM calculations of low-lying negative-parity states in ^{137}Nd and neighboring odd-proton nuclei. The parameters of the neutron dynamical fermion-boson interaction are Γ_0=0.3 MeV, χ=-1 and χ'=-0.2, and for protons: Γ_0= 0.22 MeV, χ=+1 and χ'=+0.2. For the exchange interaction $\Lambda_0^\nu = 1.0$ and $\Lambda_0^\pi = 1.5$ for neutrons and protons, respectively. The strength parameter of the pair-breaking interaction is $U_0 = 0$ and $U_2 = 0.2$ MeV, both for protons and neutrons in broken pairs. The residual interaction between unpaired fermions is a surface δ-force with strength parameters: $V_{\nu\nu} = -0.1$, $V_{\pi\pi} = -0.1$ and $V_{\nu\pi} = -0.9$ for neutron-neutron, proton-proton and proton-neutron, respectively.
In Fig. 4 we display the calculated spectrum of positive-parity states. According to the structure of wave functions, states are classified in bands labeled in such a way that a direct comparison can be made with their experimental counterparts. The calculated positive-parity structures 3, 5, 7, 8, 10 and 11, as well as the ground state band an the quasi γ-band, have to be compared with the experimental bands of Fig. 3. The bands 3, 5, and 7 result from the alignment of a pair of protons in the $h_{11/2}$ orbital. The band 8 corresponds to two $h_{11/2}$ neutrons coupled to the boson core. Finally, the two dipole bands 10 and 11 correspond to four-quasiparticle states, two protons and two neutrons in their respective $h_{11/2}$ orbitals, coupled to the ground state band of the core. The occurrence of regular dipole bands ($\Delta I = 1$) in nearly spherical and transitional nuclei presents an interesting phenomenon. In the semiclassical picture of the cranked shell model, $\Delta I = 1$ high-spin bands have been described as TAC (Tilted Axis Cranking) solutions [13]. In our model such $\Delta I = 1$ structures are produced by the fermion-boson interactions. However, in order to obtain the correct energy spacings for bands 10 and 11, it was necessary to include the additional term (3) in the boson quadrupole operator. We have also found that a crucial role in the excitation spectrum of these bands is played by the proton-neutron delta-interaction. It should be noted that bands 10 and 11 have been recently described in the framework of the projected shell model [14]. In order to obtain bands of dipole character, a permanent deformation had to be assumed, which in turn made energetically more favorable one of the neutrons to occupy the $\nu f_{7/2}$ orbital. Therefore the configuration $(\pi h_{11/2})^2\ \nu f_{7/2}\ \nu h_{11/2}$

was assigned to the bands 10 and 11. The dipole character of the bands results from the coupling of the neutron hole in $h_{11/2}$ and the neutron particle in $f_{7/2}$. We believe that this interpretation is not correct. In our calculations, structures based on the $\nu f_{7/2}$ orbital are found high above the yrast. Using a unique set of parameters, we have been able to reproduce the complete experimental spectrum of positive parity states, from the ground state band, to angular momentum 29 $\hbar$ in band 11, at more than 13 MeV excitation energy. With the same set of parameters we have also calculated negative parity states based on the neutron orbitals $s_{1/2}$, $d_{3/2}$ and $h_{11/2}$. The resulting bands reproduce the experimental data.

3.1 Conclusions

We have used an extension of the Interacting Boson Model to describe the physics of high-spin states in nuclei. Compared with traditional models based on the cranking approximation, the present approach provides several advantages. High-spin bands can be described not only in well deformed, but also in transitional and spherical nuclei. A single set of parameters and a well defined Hamiltonian are used to calculate both ground state collective bands and high-spin two- and four-quasiparticle bands. Polarization effects directly result from the model boson-fermion interactions. The model produces not only energy spectra, but also wave functions which can be used to calculate electromagnetic properties. All calculations are performed in the laboratory frame, and therefore produce results that can be directly compared with experimental data.
In the spherical nucleus ^{104}Cd the model reproduces the structure of high-spin bands built on a vibrational structure. The bands result from the alignment of a pair of neutrons in the orbitals $\nu d_{5/2}$ and $\nu g_{7/2}$. For the transitional nucleus ^{136}Nd, our calculation reproduce the complete experimental excitation spectrum of positive and negative parity states. In particular, we have been able to obtain a correct description of the two high-spin $(\pi h_{11/2})^2$ $(\nu h_{11/2})^2$ dipole bands.

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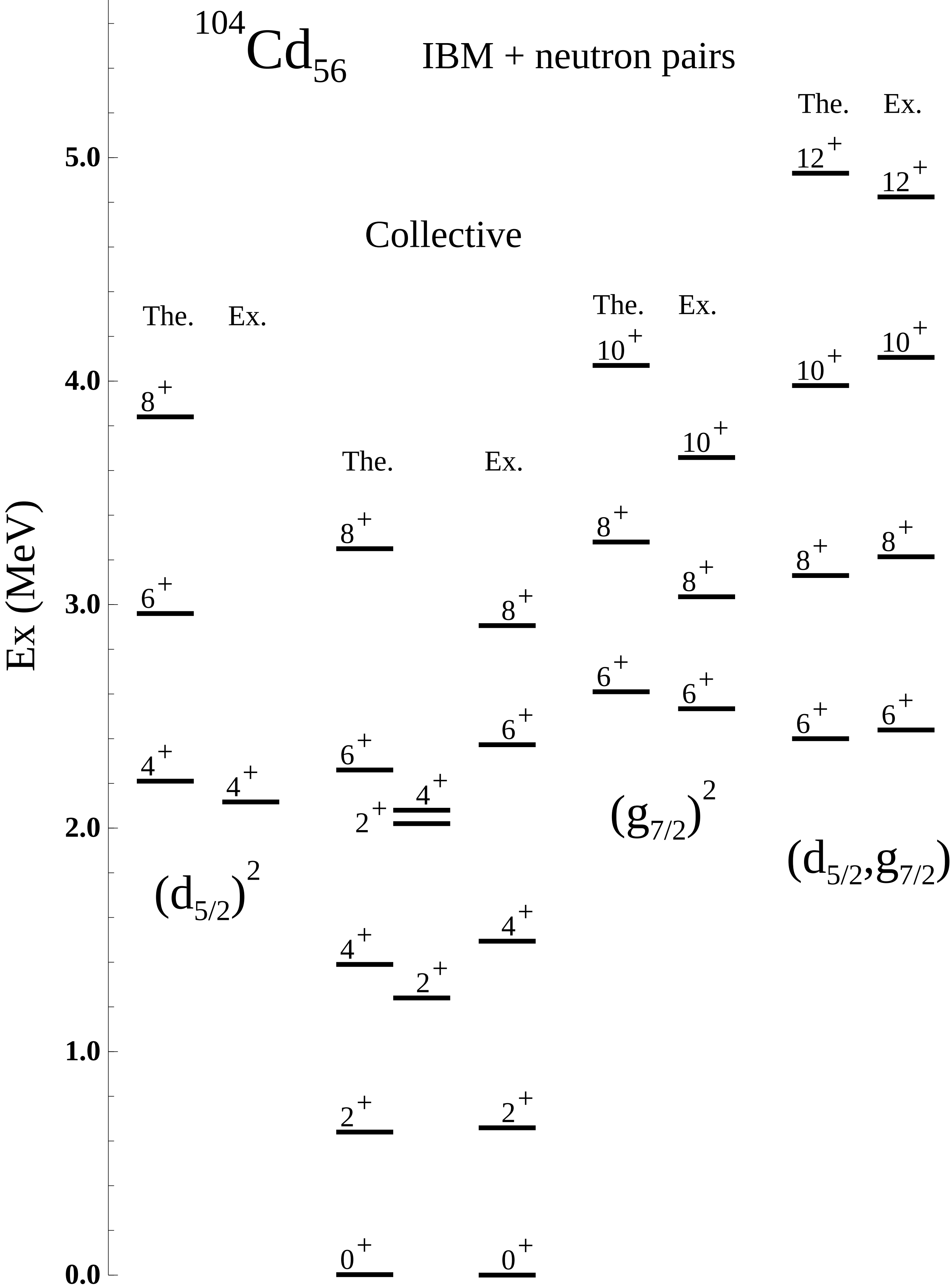
Figure Captions

FIG. 1. Results of the IBM plus broken pair calculation for positive-parity states compared with experimental levels in ^{104}Cd.

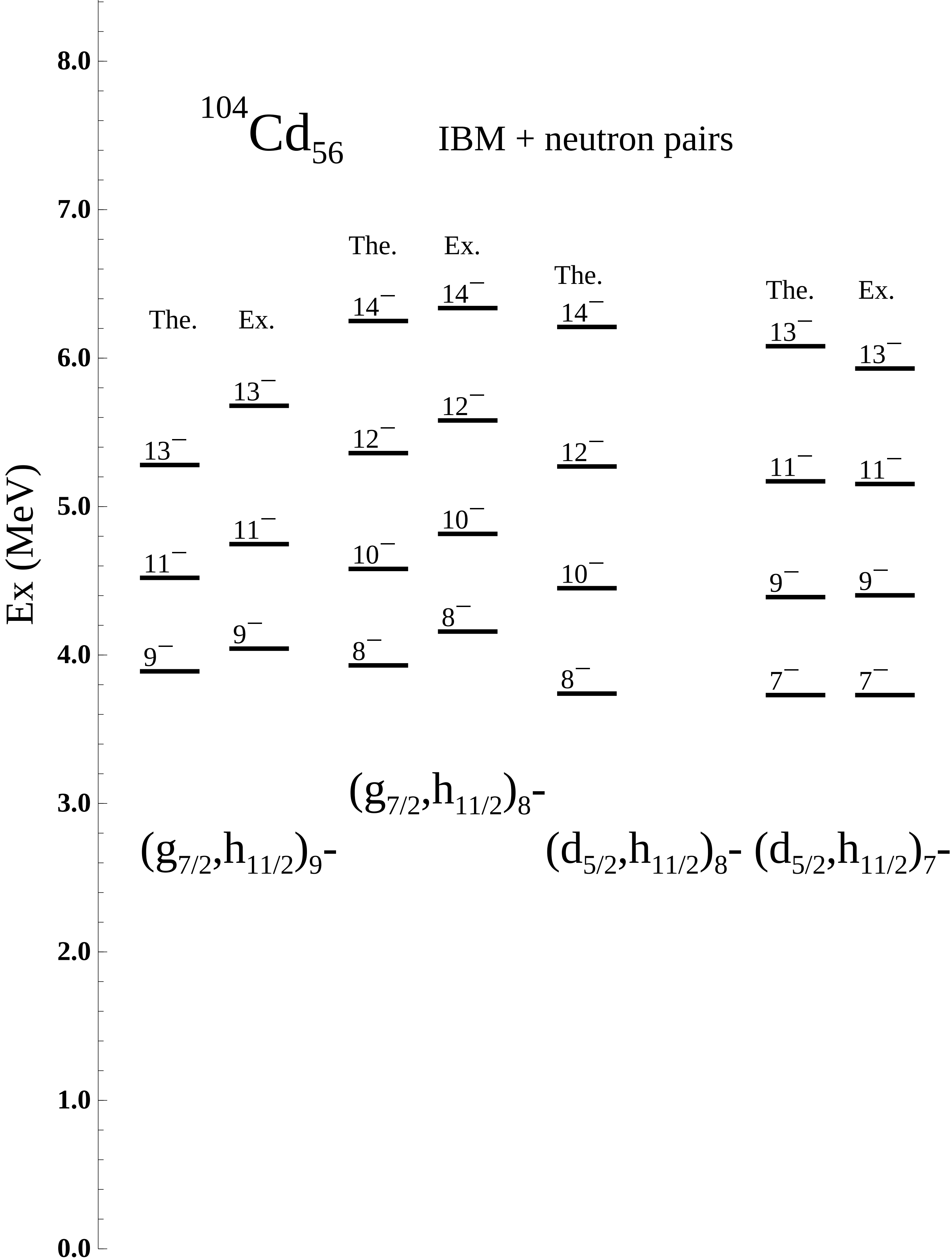
FIG. 2. Negative-parity states in ^{104}Cd compared with results of the IBM plus broken pair calculation.

FIG. 3. Experimental excitation spectrum of positive-parity states in ^{136}Nd.

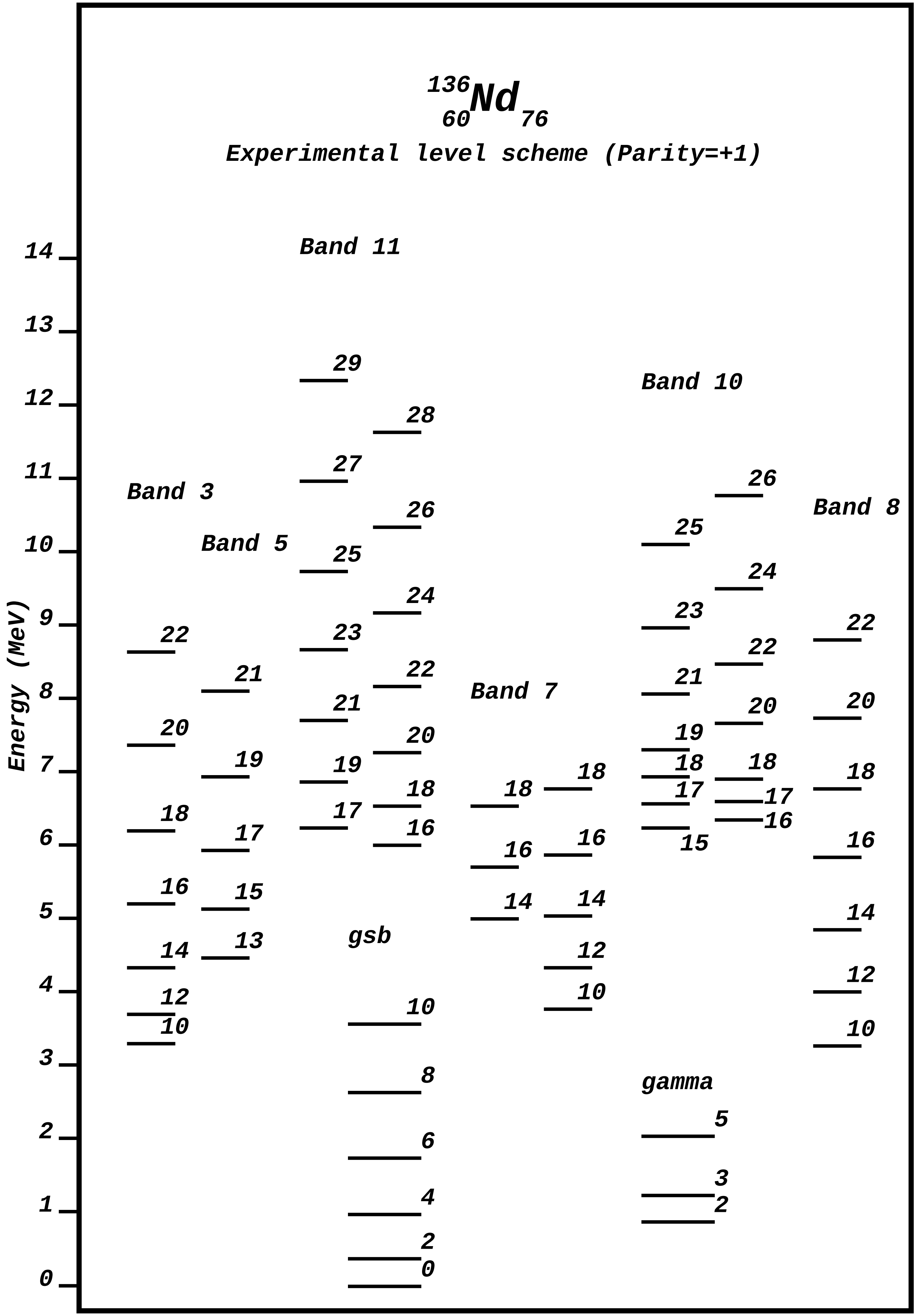
FIG. 4. Results of IBM plus broken pairs calculation for positive parity bands in ^{136}Nd



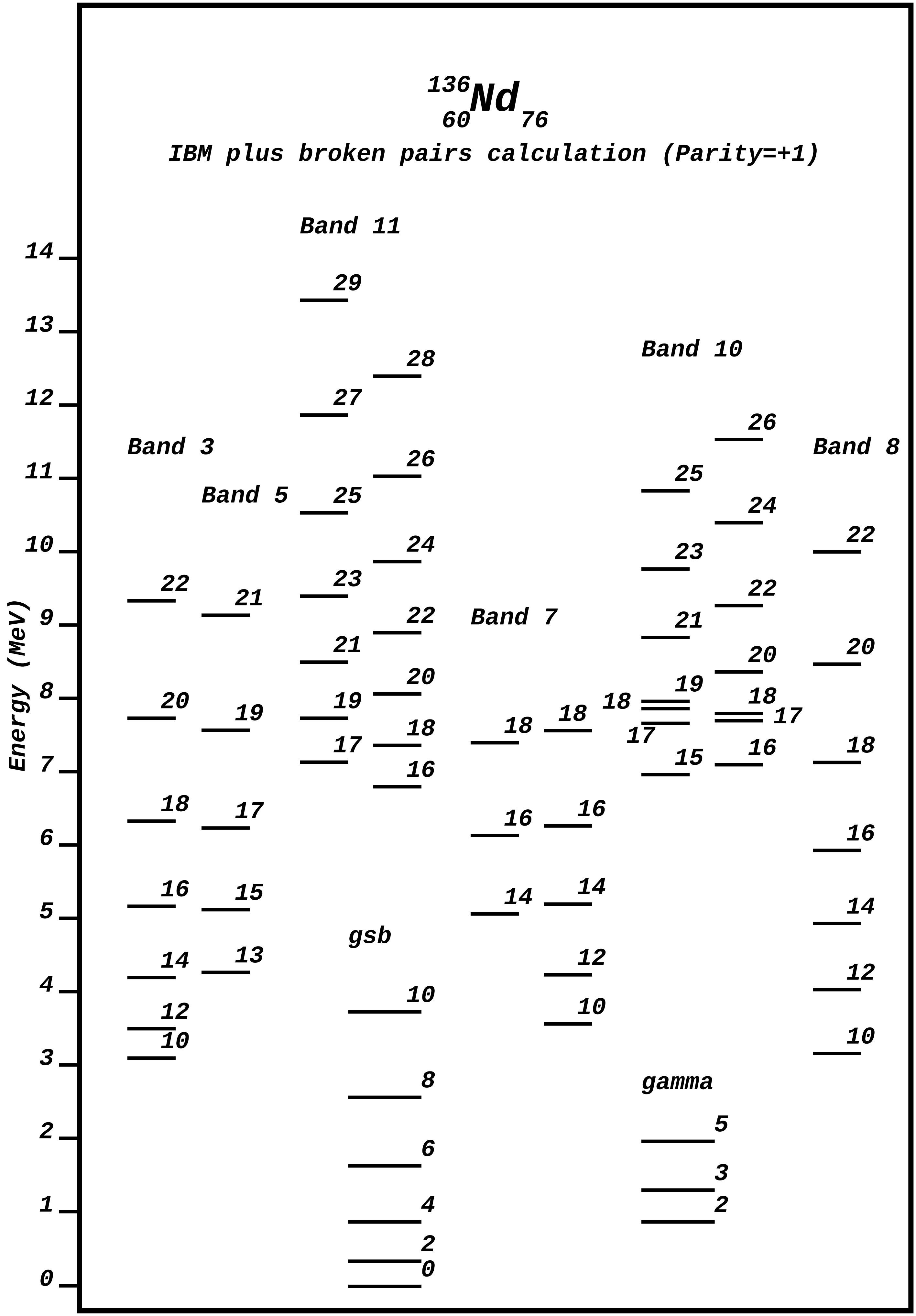
$^{104}Cd_{56}$ IBM + neutron pairs
Ex (MeV)
0.0
1.0
2.0
3.0
4.0
5.0
The.
Ex.
8^+
6^+
4^+
4^+
$(d_{5/2})^2$
Collective
The.
Ex.
8^+
6^+
4^+
2^+
4^+
2^+
2^+
0^+
8^+
6^+
4^+
2^+
0^+
The.
Ex.
10^+
8^+
6^+
10^+
8^+
6^+
$(g_{7/2})^2$
The.
Ex.
12^+
10^+
8^+
6^+
12^+
10^+
8^+
6^+
$(d_{5/2},g_{7/2})$



$^{104}Cd_{56}$
IBM + neutron pairs
Ex (MeV)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
The.
Ex.
9^-
11^-
13^-
$(g_{7/2},h_{11/2})_9$-
8^-
10^-
12^-
14^-
$(g_{7/2},h_{11/2})_8$-
$(d_{5/2},h_{11/2})_8$-
7^-
$(d_{5/2},h_{11/2})_7$-



$^{136}_{60}Nd_{76}$
Experimental level scheme (Parity=+1)
Energy (MeV)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Band 3
22
20
18
16
14
12
10
Band 5
21
19
17
15
13
Band 11
29
28
27
26
25
24
23
22
21
20
19
18
17
16
gsb
10
8
6
4
2
0
Band 7
18
16
14
18
16
14
12
10
Band 10
26
25
24
23
22
21
20
19
18
18
17
17
16
15
gamma
5
3
2
Band 8
22
20
18
16
14
12
10



136 60 Nd 76
IBM plus broken pairs calculation (Parity=+1)
Energy (MeV)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Band 11
29
28
27
26
25
24
23
22
21
20
19
18
17
16
Band 3
22
20
18
16
14
12
10
Band 5
21
19
17
15
13
gsb
10
8
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4
2
0
Band 7
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16
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Band 10
26
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23
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17
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17
15
16
Band 8
22
20
18
16
14
12
10
gamma
5
3
2